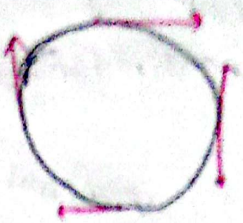
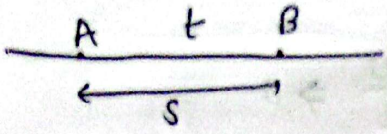
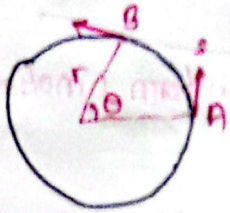
CIRCULAR MOTION



- velocity acts tangentially.



$$\boxed{V = \frac{s}{t}}$$



$$\theta = \frac{s}{r}$$

$$\boxed{s = r\theta}$$

$$\frac{ds}{dt} = \frac{r\,d\theta}{dt}$$

$$\boxed{V = r\omega}$$

lin. vel. ang. vel.

Linear paramt: s, v, a — angular parameter: θ, ω, α

$s = r\theta$

$V = r\omega$

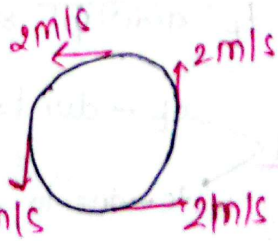
$a = r\alpha$

$$\left.\begin{aligned} v &= u + at \\ \omega &= \omega_0 + \alpha t \end{aligned}\right\} \qquad \left.\begin{aligned} v^2 &= u^2 + 2as \\ \omega^2 &= \omega_0^2 + 2\alpha\theta \end{aligned}\right\}$$

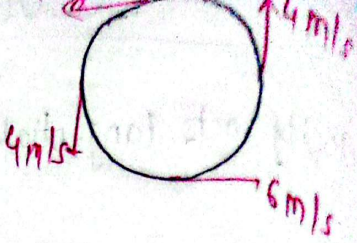
$$\left.\begin{aligned} s &= ut + \frac{1}{2}at^2 \\ \theta &= \omega_0 t + \frac{1}{2}\alpha t^2 \end{aligned}\right\} \qquad \left.\begin{aligned} F &= ma \\ \tau &= I\alpha \end{aligned}\right\}$$

$$\left.\begin{aligned} K.E_t &= \frac{1}{2}mv^2 \\ K.E_r &= \frac{1}{2}I\omega^2 \end{aligned}\right\} \qquad \left.\begin{aligned} a &= \frac{dv}{dt} \\ \alpha &= \frac{d\omega}{dt} \end{aligned}\right\}$$

→ V → magnitude — constant; Direction — changes } uniform (motion



V → magnitude → varies
V → direction → varies.
} Non uniform cir. motion



→ $\omega = \frac{v \to \text{const}}{r \to \text{const}}$ $\qquad \alpha = \frac{d\omega}{dt} \Rightarrow 0$

uniform c. motion

→ $\omega = \frac{v \to \text{change}}{r}$ $\qquad \alpha = \frac{d\omega}{dt} \neq 0$

non-uniform c motion

→ velocity → acts tangentially, F → towards centre.
↳ circular motion

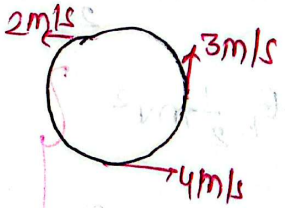
Centripetal Force or radial Force :-

$$F_c = \frac{mv^2}{r} = m\omega^2 r.$$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

→ When the particle leaves the circular track, it flies radially / tangentially.

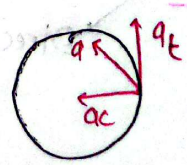
V → vary magnitude ⇒ a_t
V → vary Direction = a_c.



• If a_t & F_t नलागेको भए → Uniform C motion.

but, Non-UCM → a_t → due to change in mag. of vel.
→ a_c → due to direction change.

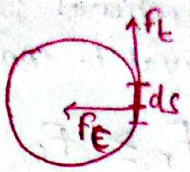
In non-UCM, $\vec{a} = \vec{a}_t + \vec{a}_c \Rightarrow \boxed{a = \sqrt{a_t^2 + a_c^2}}$



$F = \sqrt{F_t^2 + F_c^2}$.

Q In a circular motion, work is done by ---.

(a) F_t (b) F_c (c) both (d) None



- centripetal force,

$W = F_c \cdot ds \cos\theta \quad (\theta = 90^\circ)$

$W = 0$

- for tangential force,

$W = F_t \cdot ds \cos\theta \quad (\theta = 0^\circ)$

$W = F_t \cdot ds \neq 0.$

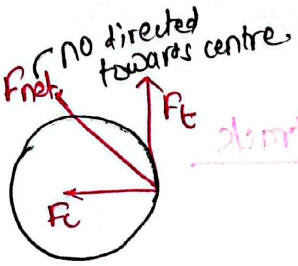
* Non UCM

a_t = non-zero

F_t = non-zero

$\alpha \rightarrow$ Non-zero

$I \rightarrow$ non-zero



F_{net}, a_{net} → non-directed towards the centre

Workdone → non-zero

UCM

a_t = zero

F_t = zero

$\alpha \rightarrow$ zero

$I \rightarrow$ zero

F_{net}, a_{net} → directed towards centre.

Workdone → zero.

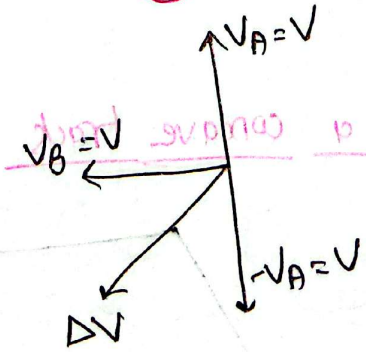
UCM

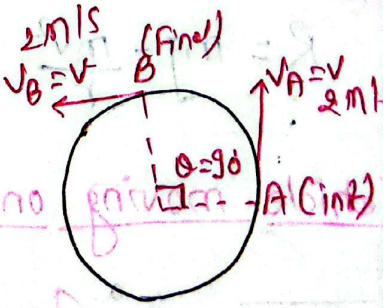
V → mag (constant) ($a_t = 0$)

V → direction (change) } only centripetal acc. (a_c)

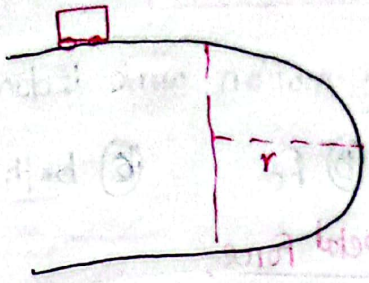
$\Delta V = \vec{V_B} - \vec{V_A}$

$\Delta V = 2V \sin\frac{\theta}{2}$





Vehicle not skidding



frictional force must be greater or equal to centripetal force.

$$f_r \geq F_c$$

$$\mu R \geq \frac{mv^2}{r} \qquad (R = mg)$$

$$\mu mg \geq \frac{mv^2}{r}$$

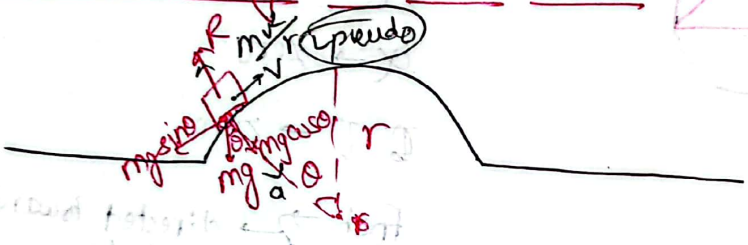
$$v^2 < \mu rg$$

$$v < \sqrt{\mu rg}$$ (skid अथवा safe अथवा)

If $v > \sqrt{\mu rg}$ ⇓ skid हुन्छ (not safe)

$$v = \sqrt{\mu rg}$$ ← max. safe speed.

Vehicle moving on a convex track



$$R + \frac{mv^2}{r} = mg\cos\theta$$

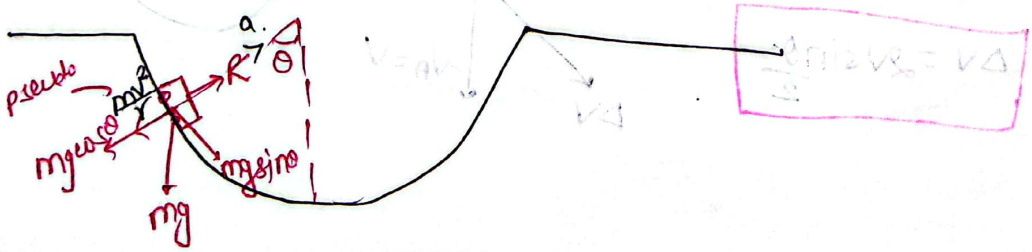
$$R = mg\cos\theta - \frac{mv^2}{r}$$

→ Normal reaction (R) goes on increasing. Maxm at top.

at top; $\theta = 0°$

$$R = mg - \frac{mv^2}{r}$$

Vehicle moving on a concave track



$$R = mg\cos\theta + \frac{mv^2}{r}$$

→ R goes on increasing. & maxm at bottom.

At bottom, $\theta = 0°$

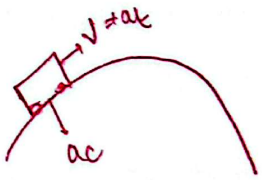
$$R = mg + \frac{mv^2}{r}$$

Q. A vehicle is speeding up on a conver circular track of radius 10m at the rate of $3m/s^2$. If the velocity at any instant is 20m/s Find the resultant acceleration?

ⓐ $3m/s^2$ ⓑ $4m/s^2$ ⓒ $5m/s^2$ ⓓ $2m/s^2$

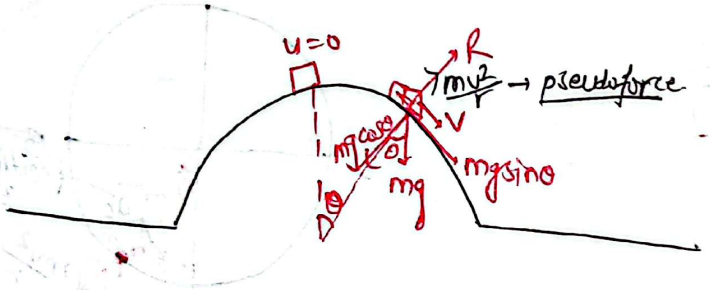
$a_t = 3m/s^2$

$a_c = \frac{v^2}{r} = \frac{20\times20}{100} = 4m/s^2.$



$a = \sqrt{a_t^2 + a_c^2} = 5m/s^2.$

*



A body at the top of hemispherical smooth surfaces falls down; what is the angle made by radius vector, when body fly off tangentially

$$R + \frac{Mv^2}{r} = mg\cos\theta$$

if body leaves the contact, $R = 0$.

$$\frac{mv^2}{r} = mg\cos\theta$$

$$\cos\theta = \frac{v^2}{rg}$$

$$\theta = \cos^{-1}\left(\frac{v^2}{rg}\right)$$

What is the vertical height from top when body fly off?

(a) $r/2$ (b) $r/3$ (c) $r/4$ (d) $3r/4$



$\cos\theta = \frac{r-h}{r}$ $\quad \cos\theta = \frac{v^2}{rg}$

$\frac{v^2}{rg} = \frac{r-h}{r}$

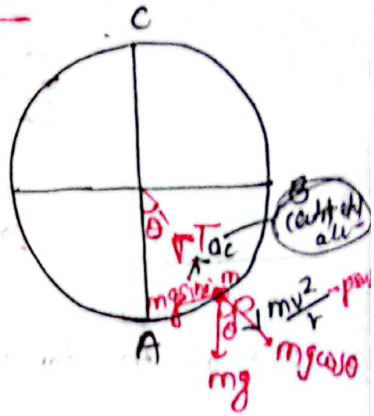
$mg - kg$ $\quad \frac{2gh}{rg} = \frac{r-h}{r}$ $\quad v = \sqrt{2gh}$



$2h = r - h$

$h = r/3$

* A mass (m) rotating in a vertical circle



• at any general point :-

$T = mg\cos\theta + \frac{mv^2}{r}$

• at bottom point A

$(\theta = 0^\circ)$

$T_A = mg + \frac{mv_A^2}{r}$

• at horizontal position B

$\theta = 90^\circ, \quad \cos 90^\circ = 0$

$T_B = \frac{mv_B^2}{r}$ $\quad (T_B = T_D)$

• at topmost point C

$\theta = 180^\circ$

$T_C = \frac{mv_C^2}{r} - mg$

So, [maxm tension → bottom. min. tension → topmost.]

To complete the vertical circle;

- minimum tension at C ; $T_c = 0$

$$\therefore \frac{mv_c^2}{r} - mg = 0$$

$$\text{or, } \frac{mv_c^2}{r} = mg$$

$$\text{or, } v_c^2 = rg$$

$$\therefore \boxed{v_c = \sqrt{rg}}$$ — at topmost point(C). min velocity.

min. Velocity at bottom :—

$(P.E)_A = 0.$

$E_A = E_C$ (energy conserved)

$(h = 2r)$

$$(K.E)_A = (P.E)_C + (K.E)_C$$

$$\text{or, } \frac{1}{2}mv_A^2 = mg2r + \frac{1}{2}mv_c^2$$

$$\therefore v_A^2 = 4rg + rg$$

$$\therefore v_A^2 = 5rg$$

$$\therefore \boxed{v_A = \sqrt{5rg}}$$

Horizontal position

$$E_A = E_B$$

$$\frac{1}{2}mv_A^2 = mgh + \frac{1}{2}mv_B^2$$

$$v_A^2 = 2gr + v_B^2$$

$$5rg = 2gr + v_B^2$$

$$3rg = v_B^2$$

$$\therefore \boxed{v_B = \sqrt{3gr}}$$

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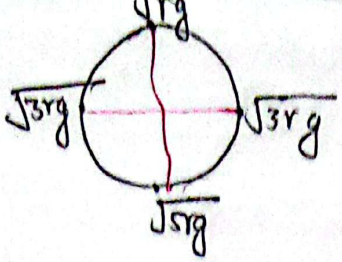


Q. $T_{max} - T_{min} = ?$

$$T_{max} = \frac{mv_A^2}{r} + mg$$

$$= \frac{m\,5rg}{r} + mg = 6mg$$

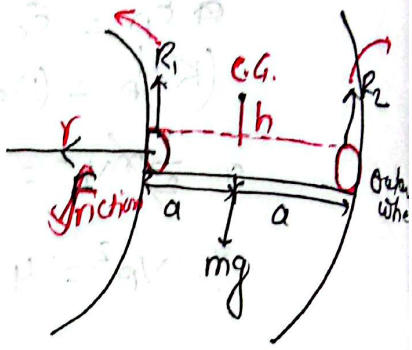
$T_{min} = 0$.

$$\therefore \boxed{T_{max} - T_{min} = 6mg}$$

Overturning of vehicle

$h \rightarrow$ height ~~from~~ of C.G.

$a \rightarrow$ semi-distance betw the wheel.



$R = mg$

$$\boxed{R_1 + R_2 = mg} \quad -(I)$$

Moment about C.G

left → outer wheel
Right → inner wheel

$$R_2 a = R_1 a + fh$$

$$(R_2 - R_1)a = f.h$$

$$(R_2 - R_1)a = \frac{mv^2}{r}h$$

$$\boxed{R_2 - R_1 = \frac{mv^2}{r}\frac{h}{a}} \quad -(II)$$

$R_2 a = R_1 a + fh$

semi dis

it provides centripetal force

Subtracting (II) from (I)

$$2R_1 = mg - \frac{mv^2}{r}\frac{h}{a}$$

$$2R_1 = \cancel{\frac{mg}{2}} \; mg\left(1 - \frac{v^2 h}{rga}\right)$$

$$R_1 = \frac{mg}{2}\left(1 - \frac{v^2 h}{rga}\right)$$

adding eqn (i) & (ii)

$$R_2 = \frac{mg}{2}\left(1 + \frac{v^2 h}{rga}\right)$$

Note:-

(1) If $R_2 > R_1$:- reaction of outer wheel > Reaction on inner wheel. (moving vehicle)

(2) If $v = 0$ (rest); $R_1 = R_2$

(3) If the speed increases; R_1 → decreases but R_2 → increases.

(4) At critical velocity, $R_1 = 0$

$\because \frac{mg}{2} \neq 0$, then, $1 - \frac{v^2 h}{rga} = 0$

$$v^2 = \frac{rga}{h}$$

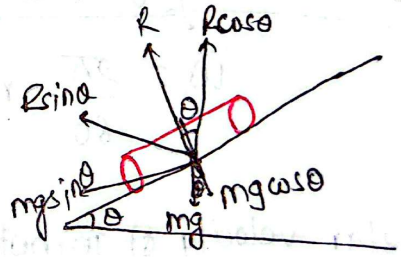
$$v = \sqrt{\frac{rga}{h}}$$

‡ Banking of road

$R\cos\theta = mg$

$R\sin\theta = \frac{mv^2}{r}$

$$\tan\theta = \frac{v^2}{rg}$$



track. And the angle of inclination?

→ $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

$= \tan^{-1}\left(\frac{20 \times 20}{20 \times 10}\right)$

$\theta = \tan^{-1}(2)$.

Q. A rod of 50m radius is banked for ~~a certain speed~~ at the correct angle for ~~certain~~ a given speed. If the speed is doubled keeping banking angle same, the radius of curvature should be :-

→ $\tan\theta = \frac{v^2}{rg}$

θ same then

$r \propto v^2$

$\therefore \frac{r_1}{r_2} = \left(\frac{v_1}{v_2}\right)^2$

$\frac{50m}{r_2} = \left(\frac{v}{2v}\right)^2$

$r_2 = 4 \times 50 = 200$ m.

Angular velocity (ω)

$\omega = \frac{2\pi}{T}$

angular velocity of second needle/hand

$\omega = \frac{2\pi}{60}$ rad/s

angular velocity of minute hand

$\omega = \frac{2\pi}{60}$ rad/min

$\omega = \frac{2\pi}{60 \times 60}$ rad/s

$\omega = \frac{2\pi}{3600}$ rad/s $= \frac{\pi}{1800}$ rad/s

• angular velocity of hour hand

$$\omega = \frac{2\pi}{12} \text{ rad/hr}$$

$$\omega = \frac{2\pi}{12 \times 60 \times 60} \text{ rad/s.}$$

$$\omega = \frac{2\pi}{43200} \text{ rad/s}$$

$$\boxed{\omega = \frac{\pi}{21600} \text{ rad/s}}$$

• angular velocity of earth

$$\omega = \frac{2\pi}{24} \text{ rad/hr}$$

$$\boxed{\omega = \frac{2\pi}{86400} \text{ rad/s} = \frac{\pi}{43200} \text{ rad/s.}}$$

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