Elasticity

- **Rigid body:-** External force applied on the body doesn't produce any deformation.

- **Elastic body:-** Body regains its original shape and size after removal of external deforming force.
  e.g. Quartz - perfect.

- **Plastic body:-** body doesnot regains its or. shape & size aft. removal of ext. def. force.
  e.g. Perfect plastic body → putty

Stress:- F/A

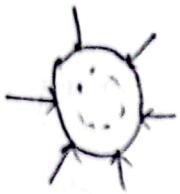


- **Normal stress:-** Normal force per unit area

- **Tangential stress:-** Tangential force applied which produces the change in shape.



- **Volumetric stress:-** Volumetric force applied which produces the change in volume.



Strain :- $\frac{\text{change}}{\text{original}}$

- **Longitudinal strain:-**

$$\frac{e}{l}$$



- **Volumetric strain**

$$= \frac{\Delta V}{V}$$



- **Shear strain or shear angle**

$$\text{shear strain} = \frac{x}{l}$$

$$\theta = x/l$$



stress ∝ strain

$\frac{\text{stress}}{\text{strain}} = E$

E

- Young modulus (Y) → for solid
- Bulk modulus (B or K) → for solid, liq & gas
- Modulus of rigidity (η) → for solid.

$Y = \frac{\text{Normal stress}}{\text{long. strain}}$

$B = \frac{\text{Normal stress}}{\text{vol strain}}$

$\eta = \frac{\text{Shear stress}}{\text{shear strain}}$

$Y = \frac{F/A}{e/l} = \frac{F.l}{e.A}$

$B = \frac{F/A}{\Delta V/V}$

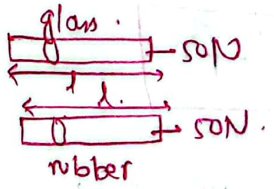
Due to change in shape.

- $Y \propto \frac{1}{\text{(extensibility)}}$
- $B \propto \frac{1}{\text{compressibility}}$

→ Glass is more elastic than rubber.

$Y = \frac{F\,A}{e\,A}$ same

$Y \propto \frac{1}{e}$



$e_{rubber} > e_{glass}$

$\therefore Y_{glass} > Y_{rubber}$

Y, B & η → depends upon the nature of material.

* Factors affecting modulus of elasticity (Y, B & η)

① effect of tempr →

Tempr ↑es → sepr between molecules ↑es

↓

intermolecular force of attraction ↓es

↓

so, elasticity ↓es. when tempr ↑es.

except in (Invar) → remains invarient with the change in tempr. (α very small)

② Adding elastic impurities → ε ↑es.

③ Adding plastic impurities → ε ↓es

→ Hammering and rolling increases elasticity.

→ Annealing → decreases elasticity.

Energy stored

→ Workdone or P.E stored.

$W = \frac{1}{2}$ force x elongation

$$W = \frac{1}{2} fe$$



IF the wire extends by l on loading mg

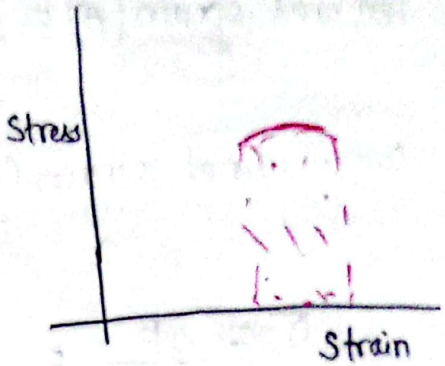
$W = \frac{1}{2}$ Force x elongation

$$W = \frac{1}{2} mgl$$

Energy density

$$\bar{\epsilon} = \frac{\text{energy stored}}{\text{volume}}$$

$$\boxed{\bar{\epsilon} = \frac{1}{2} \text{stress} \times \text{strain}}$$



Area under the stress-strain graph

↓

workdone per unit volume or energy density

.concept:-

Elastic energy stored per unit volume in fresh water of compressibility β at a depth (h) below the surface of water =?

(a) $\beta \frac{hsg}{2}$ (b) βhsg (c) $\frac{(hsg)^2}{2\beta}$ (d) $\beta \frac{(hsg)^2}{2}$

$\bar{\epsilon} = \frac{1}{2}$ stress x strain

$= \frac{1}{2} f/A \times$ strain

$= \frac{1}{2} \Delta P (\Delta P) B$

$= \frac{1}{2} (hsg)^2 \beta$

$\beta = \frac{\Delta P}{\Delta V/V}$, strain $= \frac{\Delta V}{V}$

$= \frac{\Delta P}{\beta}$

$= \Delta P \cdot \beta$

[compressibility, $\beta = \frac{1}{B}$]

$[\Delta P = hsg]$

Energy density $= \frac{1}{2}$ stress x strain

$= \frac{1}{2} \times Y$ strain x strain

$= \frac{1}{2} Y (\text{strain})^2$

$Y = \frac{\text{stress}}{\text{strain}}$

stress $= Y \times$ strain

Poisson's ratio (σ)



Lateral strain (β) = $\frac{\Delta r}{r}$ or $\frac{\Delta d}{d}$

Longitudinal strain (α) = $\frac{\Delta L}{L}$

$$\sigma = \frac{\beta}{\alpha} = -\frac{\frac{\Delta r}{r}}{\frac{\Delta l}{l}} \Rightarrow \frac{\Delta r}{r} = -\sigma\left(\frac{\Delta l}{l}\right)$$

(-ve → length ↑es, r ↓es.
r ↑es, l ↓es).

Volume of wire

$$V = \pi r^2 l$$

$$\boxed{\frac{\Delta V}{V} = 2\frac{\Delta r}{r} + \frac{\Delta l}{l}}$$

$$\frac{\Delta V}{V} = 2\left(\sigma\frac{\Delta l}{l}\right) + \frac{\Delta l}{l}$$

$$\frac{\Delta V}{V} = (-2\sigma + 1)\frac{\Delta l}{l}$$

$$\boxed{\frac{\Delta V}{V} = (1 - 2\sigma)\frac{\Delta l}{l}}$$

When $\Delta V = 0$;

$$(1 - 2\sigma)\frac{\Delta l}{l} = 0$$

Since, $\frac{\Delta l}{l} \neq 0$

So, $1 - 2\sigma = 0$

$\therefore$ $\boxed{\sigma = \frac{1}{2} = 0.5}$

σ → Theoritically → -1 to 0.5
practically → 0 to 0.5

Concept :-

If $\sigma = 0.2$ & $\frac{\Delta l}{l} = 2 \times 10^{-3}$. What is the % change in volume?

→ $\frac{\Delta V}{V} = \% = ?$

$$\frac{\Delta V}{V} = (1 - 2 \times 0.2) \times 2 \times 10^{-3}$$

$$= 0.6 \times 2 \times 10^{-3}$$

$$= 1.2 \times 10^{-3} \times 100\%$$

$$= 0.12\%$$

Note :-

Y for perfectly elastic body → ∞ $\left\{ Y = \frac{F \cdot l}{A \cdot \Delta l} = \infty \right\}$

Y for perfectly plastic body → 0

Y for rigid body → ∞

- Shear modulus (η) → only for solid
- Elasticity is due to cohesive force. (intermolecular)

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