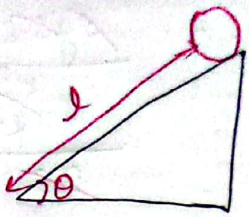
For same inclined length 'l' and same θ

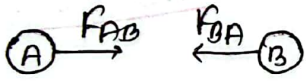


- Ring → $k^2/r^2 = 1 \Rightarrow t_{max}$
- disc → $k^2/r^2 = 1/2$
- Solid sphere → $k^2/r^2 = 2/5 \Rightarrow t_{min}$.
- Hollow sphere → $k^2/r^2 = 2/3$

$t \propto \sqrt{1 + k^2/r^2}$

Gravitation

- Gravitational force ← attractive, central force, conservative } (independent to the medium)



$F_{AB} = -F_{BA}$

→ action and reaction ⇒ Newton's third law.

→ Weakest and long range force. (Nuclear → strongest & short ra

G → constant everywhere
→ depends upon system of unit. (SI or CGS)
- SI → $6.67 \times 10^{-11}\ N m^2 kg^{-2}$
- CGS → 6.67×10^{-8} dyne $cm^2 gm^{-2}$

Q. A mass (M) is broken down into 2 smaller fragments m & M-m and then placed at certain distance apart. What is 'm' so that force between them is maximum?

→ (a) $\frac{3}{4}M$ (b) $\frac{M}{2}$ (c) $\frac{M}{4}$ (d) $\frac{M}{3}$

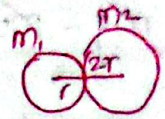
For maximum force, mass becomes (m) — r — (M-m)

half of the initial.

$F \propto m_1 m_2$

Q. Two bodies of radius r and 2r are placed [illegible] What is the relation of gravitational force with ~~distance~~ radius between them?

(a) $F \propto r^{-2}$ (b) $F \propto r^2$ (c) $F \propto r$ ~~(d)~~ $F \propto r^4$



$$F \propto \frac{m_1 m_2}{(3r)^2} \propto \frac{V_1 s_1 . V_2 s_2}{9r^2} \propto \frac{\frac{4}{3}\pi r^3 s . \frac{4}{3}\pi r^3 s}{9r^2} \propto \frac{r^6}{r^2} \propto r^4$$

$\therefore$ $\boxed{F \propto r^4}$

#Gravity:- Attractive force of planet towards its centre for its surrounding objects.

→ Gravity gives weight of body i.e. $W = mg$.

→ unit of Gravity is Newton

→ Gravity is vector quantity ⇒ direction is towards the centre of planet.

→ Cloud around the earth is due to gravity.



$$F = \frac{GMm}{R^2} = mg$$

$$\Rightarrow \boxed{g = \frac{GM}{R^2}} \leftarrow \text{acc}^r \text{ due to gravity}$$

- 'g' → depends on mass of planet, radius of planet, but doesnot depends on shape, size and mass of body.

e.g. In vacuum (no resisting force), iron ball & Feather if dropped then both falls simultaneously.

* Factors affecting 'g'

① Due to shape:-

$R_{eq} > R_{pole}$ $\therefore g \propto \frac{1}{R^2}$

$\therefore g_{pole} > g_{equator}$

$R_{eq} - R_{pole} = 21 km$

$g_{pole} \Rightarrow 9.83 m/s^2$

$g_{equator} \Rightarrow 9.78 m/s^2$

$g_{pole} - g_{equator} = 0.018 m/s^2$



② Due to rotation:-

$\cancel{m}g' = \cancel{m}g - \cancel{m}\omega^2 r \cos\theta$ $\quad \because \cos\theta = \frac{r}{R}$

$g' = g - \omega^2 r \cos\theta$ $\quad r = R\cos\theta$

$$\boxed{g' = g - \omega^2 R\cos^2\theta}$$

→ due to rotation of earth, value of g decreases.

At pole

$\theta = 90°$, $\cos^2 90 = 0$.

So, $\boxed{g'_p = g}$ → no effect to the value of g at pole due to rotation.

At equator

$\theta = 0°$, $\cos^2 0 = 1$

$\boxed{g'_{eq} = g - \omega^2 R}$ $\quad g_p - g_{eq} = \omega^2 R$ $\quad \omega = \frac{2\pi}{T}$ (24 hr)

$= 0.034 m/s^2$

Net, decrease in g at equator = $\overset{\text{rot.}}{0.034} + \overset{\text{shape}}{0.018}$

$= 0.052 m/s^2$

Q. When earth suddenly stops rotatioting, the weight of body?

(a) ↑es (b) ↓es (c) unchange (d) zero

$g' = g - \omega^2 R\cos\theta$

when stop ($\omega = 0$)

so, g' ↑es.

Q. What is the angular speed of the earth so that [illegible] equator fly off?

$$g' = g - \omega^2 R \quad (\theta = 0)$$

to fly off, $g' = 0$

$$0 = g - \omega^2 R$$

$$\omega = \sqrt{\frac{g}{R}} = \sqrt{\frac{10}{6400 \times 10^3}} = \sqrt{\frac{1}{640000}} = \frac{1}{800} \text{ rad/s}$$

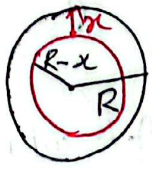
17 times more than that of present angular speed.

Due to rotation, g decreases.

Due to Height / Depth :-

→ ~~Due to depth,~~

$$g = \frac{GM}{R^2} = \frac{GV\rho}{R^2} = \frac{G\frac{4}{3}\pi R^3 \rho}{R^2}$$



$$= \frac{4}{3}\pi R \rho G$$

$$g' = \frac{GM'}{(R-x)^2} = \frac{G\frac{4}{3}\pi(R-x)^3\rho}{(R-x)^2} = \frac{4}{3}\pi(R-x)\rho G$$

$$\frac{g}{g'} = \frac{R}{(R-x)}$$

$$\boxed{g' = g\left(1 - \frac{x}{R}\right)}$$ where, x is depth from surface.

So, as depth increases from surface (x ↑) then g ↓.

. At centre, $x = R$, $\boxed{g' = 0}$

→ absolute decrease in g;

$$g - g' = \frac{2xg}{R}$$

$$\therefore \boxed{\Delta g = \frac{x}{R} g}$$

→ Fractional decrease in g

$$\frac{\Delta g}{g} = \frac{x}{R}$$

→ % decrease in g

$$\frac{\Delta g}{g} \times 100\% = \frac{x}{R} \times 100\%$$

Q. At what depth from surface the weight of man becomes half of the surface?

→

$$g' = g\left(1 - \frac{x}{R}\right)$$

$$\boxed{W' = \frac{W}{2}}$$

$$mg' = mg\left(1 - \frac{x}{R}\right)$$

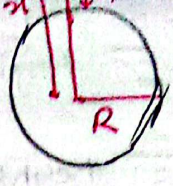
$$\frac{mg}{2} = mg\left(1 - \frac{x}{R}\right)$$

$$\frac{1}{2} = 1 - \frac{x}{R}$$

$$\frac{x}{R} = \frac{1}{2}$$

$$\boxed{x = R/2}$$

If Due to height



$$g = \frac{GM}{R^2}$$

$$g' = \frac{GM}{(R+h)^2}$$

$$\frac{g'}{g} = \frac{R^2}{(R+h)^2} = \frac{1}{\left(1+\frac{h}{R}\right)^2}$$

$$\boxed{\frac{g'}{g} = \frac{1}{\left(1+\frac{h}{R}\right)^2}}$$

$$\boxed{g' = g\left(\frac{R}{R+h}\right)^2}$$

$$g' = g\left(1+\frac{h}{R}\right)^{-2}$$

$$\therefore \boxed{g' = g\left(1-\frac{2h}{R}\right)}$$

→ As height ↑es from surface, g' goes on decreasing.

Q. At what height from surface, weight of man becomes half of the weight at surface?

$$g' = g\left(\frac{R}{R+h}\right)^2$$

$$mg' = mg\left(\frac{R}{R+h}\right)^2$$

$$\frac{mg}{2} = mg\left(\frac{R}{R+h}\right)^2$$

$$\frac{1}{2} = \left(\frac{R}{R+h}\right)^2$$

$$\text{or } (R+h)^2 = 2R^2$$

$$\text{or } R+h = \sqrt{2}R$$

$$h = \sqrt{2}R - R$$

$$\boxed{h = (\sqrt{2}-1)R}$$

$$= 0.41 \times 6400$$

$$=$$

→ Surface मा g. छ

$g/2$ हुने height ⇒ $h = (\sqrt{2}-1)R$

$g/3$ हुने height ⇒ $h = (\sqrt{3}-1)R$

∴

g/n हुने height → $h = (\sqrt{n}-1)R$

Q. At what height from surface, the value of 'g' becomes 1% of the surface?

→

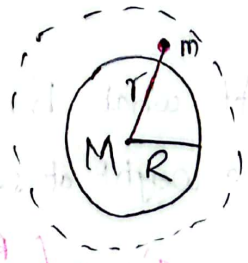
$g' = 1\%\ of\ g = \frac{g}{100} \rightarrow n = 100$

∴ $h = (\sqrt{100}-1)R$

$= 9R$

Gravitational Field Intensity

$$I = \frac{f}{m} = \frac{\frac{GMm}{r^2}}{m} = \frac{GM}{r^2}$$



∴ $I = \frac{GM}{r^2}$ $\quad r = R+h$

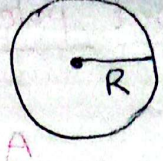
At surface, $r = R$.

$$I = \frac{GM}{R^2} = g$$

- I is vector → direction towards the centre.

Unit → N/m or m/s^2

for hollow sphere



$I_{inside} = 0$ (no mass)

$I_{surface} = \frac{GM}{R^2}$

$I_{Outside} = \frac{GM}{r^2}$ $\therefore$ $\boxed{r>R}$

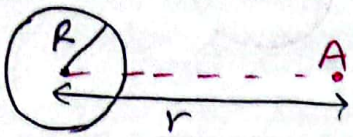
Gravital field int. For Hollow sphere	elect field inter For spherical charged conductor	Mag field int. For Hollow cylindrical rod
$I_{inside} = 0$	$E_{inside} = 0$	$B_{inside} = 0$
$I_{surface} = \frac{GM}{R^2}$	$E_{surface} = \frac{KQ}{R^2}$	$B_{surface} = \frac{\mu_0 I}{2\pi R}$
$I_{outside} = \frac{GM}{r^2}$ $(r>R)$	$E_{outside} = \frac{KQ}{r^2}$ $(r>R)$	$B_{outside} = \frac{\mu_0 I}{2\pi r}$ $(r>R.$



For solid sphere

Grav. field inter. Solid sphere	elect field inter. Solid non-conductive sphere	Mag. field inter. Solid cylin rod.
$I_{inside} = \frac{GMr}{R^3}$ $(r<R)$	$E_{inside} = \frac{KQr}{R^3}$ $(r<R)$	$B_{inside} = \frac{\mu_0 I r}{2\pi R^2}$
$I_{ins.} \propto r$	$E_{ins} \propto r$	$B_{ins} \propto r$
$I_{centre} = 0$	$E_{centre} = 0$	$B_{centre} = 0$
$I_{surface} = \frac{GM}{R^2}$	$E_{surface} = \frac{KQ}{R^2}$	$B_{surface} = \frac{\mu_0 I}{2\pi R}$
$I_{outside} = \frac{GM}{r^2}$ $(r>R)$	$E_{outsi.} = \frac{KQ}{r^2}$ $(r>R)$	$B_{outs} = \frac{\mu_0 I}{2\pi r}$ $(r>$

Gravitational potential



m
At ∞

$$V_A = \frac{W_{\infty A}}{m}$$

$$\boxed{V_A = -\frac{GM}{r}}$$

For hollow sphere

$$V_{inside} = V_{surface} = -\frac{GM}{R}$$

$$V_{outside} = -\frac{GM}{r} \quad (r>R)$$

For spherical charged conductor

$$V_{inside} = V_{surface} = \frac{KQ}{R}$$

For solid sphere

$$V_{inside} = \frac{GM}{2R^3}(3R^2 - r^2) \quad (r<R)$$

$$V_{centre} = \frac{3}{2}\frac{GM}{R} \quad (r=0)$$

$$V_{surface} = \frac{GM}{R} \quad (r=R)$$

$$V_{outside} = \frac{GM}{r} \quad (r>R)$$

For solid non-conducting sp

$$V_{inside} = \frac{KQ}{2R^3}(3R^2 - r^2) \quad (r<$$

$$V_{centre} = \frac{3}{2}\frac{KQ}{R} \quad (r=0)$$

$$V_{surface} = \frac{KQ}{R} \quad (r=R)$$

$$V_{outside} = \frac{KQ}{r} \quad (r>R)$$

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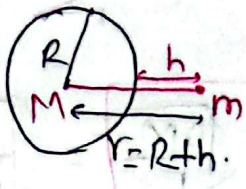
$V_{centre} = \frac{3}{2}\frac{GM}{R} - V_{surface}$

$$V_{centre} = \frac{3}{2} V_{surface}$$

Gravitational potential energy

$$U = mV \qquad V = -\frac{GM}{r}$$



$$\therefore U = -\frac{GMm}{r}$$

At surface, $h = 0$. $\therefore R = r$

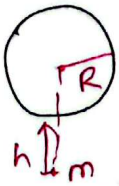
$$U_{surface} = -\frac{GMm}{R}$$

At infinity:

$$U = 0$$

Q. # change in Gravitational potential energy for a mass 'm' is rest from surface of earth to the height (h) from surface = ?

$$U_{surface} = -\frac{GMm}{R}$$



$$U_{height} = -\frac{GMm}{R+h}$$

$$g = \frac{GM}{R^2} \Rightarrow GM = gR^2$$

$$\Delta U = -\frac{GMm}{R+h} + \frac{GMm}{R} = -GMm\left(\frac{R-R-h}{R(R+h)}\right)$$

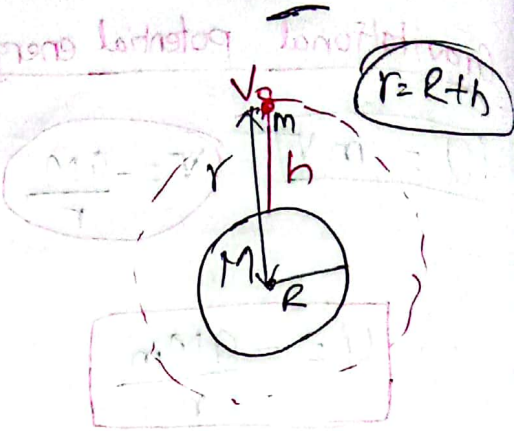
$$= -GMm\left(\frac{1}{R+h} - \frac{1}{R}\right) = \frac{GMm.h}{R(R+h)} = \frac{gR^2mh}{R(R+h)}$$

$$\therefore \Delta U = \frac{mghR}{R+h}$$

$$\Delta U = mgh\frac{R}{R+h}$$

Motion of satellite

$$\frac{mv_0^2}{r} = \frac{GMm}{r^2}$$



$$V_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR^2}{R+h}}$$

$V_0 \rightarrow$ orbital velocity

At close to the surface, $h \approx 0$.

$$V_0 = \sqrt{gR} = 8\,km/s$$

* angular speed (ω)

$$\omega = \frac{V_0}{r} = \sqrt{\frac{GM}{r^3}}$$

If $r = R \Rightarrow$ close to the surface.

$$\omega = \sqrt{\frac{GM}{R^3}} = \sqrt{\frac{gR^2}{R^3}} = \sqrt{\frac{g}{R}} = \frac{1}{800}\ rad/s$$

$$\omega = \frac{1}{800}\ rad/s$$

* Time period (T)

$$T = \frac{2\pi r}{V_0} = \frac{2\pi r}{\sqrt{\frac{GM}{r}}} = \frac{2\pi}{\sqrt{\frac{GM}{r^3}}}$$

$$T \propto \sqrt{r^3}$$

$$T^2 \propto r^3$$ — Kepler's law.

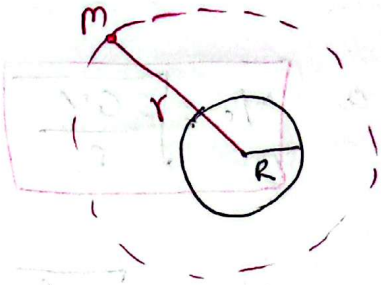
close to the surface,

$$T = \frac{2\pi}{\sqrt{\frac{GR^2}{R^3}}} = 2\pi\sqrt{\frac{R}{g}} = 84.6\,\text{min}$$

$$\therefore \boxed{T = 84.6\,\text{min}}$$

* Close to the surface

- $V_0 = 8\,\text{km/s}$ → 1st cosmic vel.
- $\omega = \frac{1}{800}$ rad/s
- $T = 84.6\,\text{min}$



$$KE = \frac{1}{2}mV_0^2 = \frac{1}{2}m\frac{GM}{r} = \frac{GMm}{2r}$$

$$\therefore \boxed{K.E = \frac{GMm}{2(R+h)}}$$

$$\boxed{P.E = -\frac{GMm}{r}}$$

$$\boxed{T.E = K.E + P.E. = -\frac{GMm}{2r}}$$

$$\boxed{K.E = x, \quad T.E = -x. \quad P.E = -2x.}$$

$$\boxed{T.E = -K.E = 2.P.E}$$

Escape Velocity



Total energy at surface = (T.E) at ∞

$(K.E + P.E)_{at\ surf} = 0 + 0$ $\quad (K.E = P.E = 0)$

$$\frac{1}{2}mv_e^2 + 0 - \frac{GMm}{R} = 0$$

or, $$\frac{1}{2}mv_e^2 = \frac{GMm}{R}$$

$$\therefore \boxed{V_e = \sqrt{\frac{2GM}{R}}}$$ V_e = escape velocity

$$\therefore V_e = \sqrt{\frac{2gR^2}{R}} = \sqrt{2gR} = 11.2\ km/s$$

→ V_e doesnot depend on the angle of projection.

$$V_e = \sqrt{2gR} \quad , \quad V_0 = \sqrt{gR}$$

By what, % escape velocity is greater than the orbital velocity?

$$\% = \frac{V_e - V_0}{V_0} \times 100\%$$

$$= \frac{\sqrt{2gr} - \sqrt{gr}}{\sqrt{gr}} \times 100\%$$

$$= (\sqrt{2} - 1) \times 100\%$$

$$= 0.41 \times 100.1$$

$$\approx 41.\%$$

Q. What additional K.E is required to escape a body [illegible] close to the surface of earth?

(a) 41% (b) 100% (c) 25% (d) 75%

(a) E (b) 2E (c) E/2 (d) E/4

$$K.E = \frac{1}{2} m v_0^2 = E$$

$$(K.E)_{esc} = \frac{1}{2} m v_e^2$$

$$= \frac{1}{2} m 2 v_0^2$$

$$= 2E$$

$(2E - E) = (E)$ additional

⇒ If v is the velocity of projection of a body,

$v < \sqrt{\frac{2GM}{R}}$ i.e. $v < v_e$ → parabolic path (returns to the surface of earth).

$v = \sqrt{\frac{GM}{R}}$ i.e. $v = v_0$ → circular path.

surface नजिकको orbital

$v > \sqrt{\frac{2GM}{R}}$ i.e. $v > v_e$ → path becomes hyperbolic.

⇒ If $v = v_e$ ⇓ body just escape हुन्छ finally, infinity मा K.E zero हुन्छ।

$v > v_e$ ⇓ escape हुन्छ र ∞ मा (free space मा) पुरोर ($v \neq 0$) हुन्छ।

IF $v > v_e$

$$(T.E)_{\text{at surface}} = (T.E)_{\text{at}\infty}$$

$$K.E_{\text{at surf}} + P.E_{\text{at surf}} = K.E_{\text{at}\infty} + P.E_{\text{at}\infty}$$

$$\frac{1}{2}mv^2 + \left(\frac{-GMm}{R}\right) = \frac{1}{2}mv'^2 + 0$$

$$\frac{1}{2}v^2 - \frac{GM}{R} = \frac{1}{2}v'^2$$

$$v^2 - \frac{2GM}{R} = v'^2 \quad (+ve)$$

$$\therefore \boxed{v^2 - v_e^2 = v'^2}$$

IF $v = nv_e$

$$(nv_e)^2 - v_e^2 = v'^2$$

$$v'^2 = (n^2-1)v_e^2$$

$$\therefore \boxed{v' = \sqrt{n^2-1}\, v_e}$$

Q. If the body is projected with the speed double of the escape velocity of earth, the velocity at free space = ?

→ $v' = \sqrt{2^2-1}\, v_e$ ($v = 2v_e$)

$$\boxed{v' = \sqrt{3}\, v_e}$$

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