Hydrostatics

Density of body = $\frac{\text{Mass of body}}{\text{Volume occupied by the body}}$

For Hollow body →

- $V_{body} > V_{substance}$
- $\rho_{body} < \rho_{substance}$.



Relative density

$$R.D = \frac{\rho_{body}}{\rho_{water}}$$ water at 4°C

m_1 = mass of liquid of density ρ_1
m_2 = mass of liquid of density ρ_2 } are mixed

Then,

$$m = m_1 + m_2$$

$$V = V_1 + V_2$$

$$= \frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}$$

$$\therefore \boxed{\rho = \frac{m}{V} = \frac{m_1 + m_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}} = \frac{\sum m_i}{\sum \frac{m_i}{\rho_i}}}$$

If $m_1 = m_2 = m$,

$$\boxed{\rho_{mixture} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}} \quad (H.M)$$

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= Volume of liq. of density ρ_1

V_2 = " " " " ρ_2.

$V = V_1 + V_2$

$m = \rho_1 V_1 + \rho_2 V_2$

$$\rho = \frac{m}{V} = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}$$

equal mass →

$$\rho_{mix} = \frac{2\rho_1 \rho_2}{\rho_1 + \rho_2}$$

equal volume →

$$\rho_{mix} = \frac{\rho_1 + \rho_2}{2}$$

If $V_1 = V_2 = V$

$$\rho = \frac{\rho_1 + \rho_2}{2} \quad \text{(A.M)}$$

→ Temp↑es → Volume↑es so, ρ↓es.

(mass → constant)

$$\frac{\rho}{\rho_0} = \frac{m/V}{m/V_0} = \frac{V_0}{V} = \frac{V_0}{V_0(1+\gamma\Delta\theta)}$$

$$\left[V = V_0(1+\gamma\Delta\theta)\right]$$

$$\therefore \rho = \frac{\rho_0}{(1+\gamma\Delta\theta)}$$

γ → coeff of cubical expansion

$$\therefore \rho = \rho_0(1-\gamma\Delta\theta)$$

→ with the ↑es in pressure → V↓es → ρ↑es.

$$\frac{\rho}{\rho_0} = \frac{V_0}{V}$$

$$B = -\frac{\Delta P}{\Delta V} \cdot V_0$$

where 'B' is Bulk modulus of elastic

$$\Delta V = -\frac{\Delta P . V_0}{B}$$

$$V - V_0 = -V_0 . \frac{\Delta P}{B}$$

$$\boxed{V = V_0\left(1 - \frac{\Delta P}{B}\right)}$$

$$\frac{\rho_0}{\rho} = \left(1 - \frac{\Delta P}{B}\right)$$

$$\boxed{\rho = \rho_0\left(1 - \frac{\Delta P}{B}\right)^{-1}}$$

$$\therefore \boxed{\rho = \rho_0\left(1 + \frac{\Delta P}{B}\right)}$$

Conclusion

$V = V_0(1 + \beta\gamma\Delta\theta)$

$\rho = \rho_0(1 - \gamma\Delta\theta)$

$V = V_0\left(1 - \frac{\Delta P}{B}\right)$

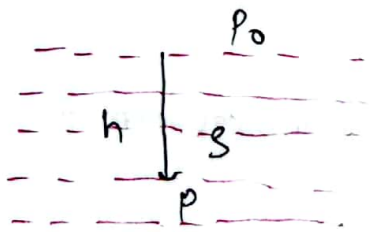
$\rho = \rho_0\left(1 + \frac{\Delta P}{B}\right)$

Pressure

$\boxed{P = \frac{F}{A}}$ → pressure has no specific direction.

↓

pressure is scalar (Tensor).



Hydrostatic pressure

$P = P_0 + h\rho g$

$\boxed{P - P_0 = h\rho g}$

Gauge pressure

1 atm pressure = 1.01×10^5 Nm
or Pasc

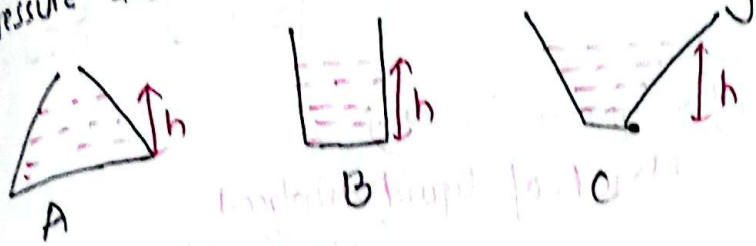
= 1.01 bar

= 760 mm of Hg or T

(Torr = mm Hg)



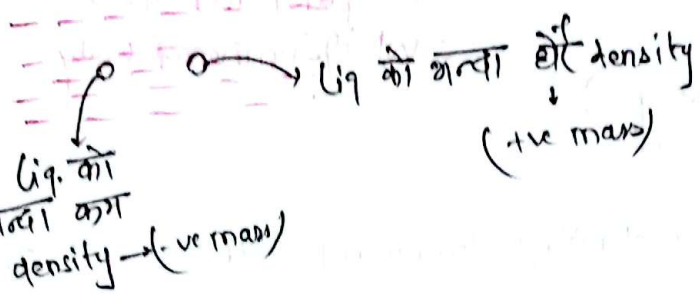
pressure act normal to the fluid boundary.



$P_A = P_B = P_C$

Depth of liquid from surface, nature of liquid, and g. but doesnot depend upon the amount of liquid & shape of vessel.





$$P = P_0 + hsg$$

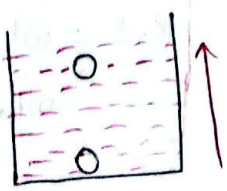
If P_0 changes to ΔP_0

$\Delta P = \Delta P_0 + \Delta(hsg)$; $\Delta(hsg) \to 0$ hsg is constant for given point

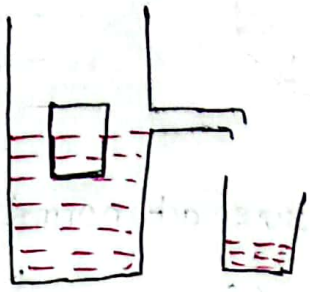
$\Delta P = \Delta P_0$ ← pressure is equally transmitted.
↳ Pascal's law

Pas

$PV = \text{constant}$



Archimedes principle



W_a = weight of body in air

W_{liq} ⇒ weight of body in liquid.

$W_a - W_{liq}$ = loss of weight in liquid
↳ wt. of liquid displaced
↳ upthrust

Upthrust (U) = wt. of liq. displaced

$$U = mg = V\sigma g$$

σ = density of liquid.

vol. of displaced liquid = vol. of immersed body.

$U \propto g$

→ Lift is accelerating downward with 'a'

$$U = V_{in}\sigma(g-a)$$

upward

$$U = V_{in}\sigma(g+a)$$

In free fall, $U=0$ $(a=g)$

Relative Density

$$R.D = \frac{\rho_{body}}{\rho_{water}}$$

$$R.D = \frac{V\rho_b g}{V\rho_w g} = \frac{\text{wt. of body}}{\text{wt. of water}}$$

$$R.D = \frac{\text{wt of body}}{\text{wt. of equal volume of water}}$$

$$R.D = \frac{\text{wt. of body}}{\text{upthrust of water}}$$

$$R.D = \frac{\text{wt. of body}}{\text{loss of wt. of body in water}}$$

$$R.D = \frac{W_b}{W_a - W_b}$$

Concept →

Loss of wt. of body in water = a

Loss of wt. of body in liquid = b

$U_w = V\sigma_w g = a$

$U_{liq} = V\sigma_{liq} g = b$

$$\frac{\sigma_{liq}}{\sigma_w} = \frac{U_{liq}}{U_w} = \frac{\text{loss of wt in liquid}}{\text{Loss of wt. in water}}$$

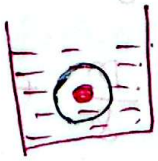
$$\frac{b}{a} = \frac{W_a - W_{liq}}{W_a - W_w}$$

*Volume of cavity in a body :-



Q. A piece of copper having internal cavity weighs 264 gm in air and 221 gm in water. Find the volume of cavity? (S of copper = 8.8 gm/cc)

→



$U = V\sigma g$ V → cavity सहित को volume.

$$V = \frac{U}{\sigma g} = \frac{(264-221)g}{1 \times g} = 43 \text{ cc.}$$

Mass of body = 264 gm

Density of material = 8.8 gm/cc.

V_0 = mass of body / density of material.

cavity वाहेक को volume

$= \frac{264}{8.8} = 30\,cc.$

∴ Volume of cavity $= V - V_0$

$= 43 - 30$

$= 13\,cc.$

* Amount of impurity in the given metal

Q. A piece of brass (Zn + Cu) weighs 12 gm in air and 11 gm in water. What is the mass of copper contained in alloy? (sp. gravity of Cu & Zn are 8 & 7)

→ Alloy has mass 'm' for copper.

(12 − m) for zinc.

M → $M_{Cu} = m$, $M_{Zn} = M - m$

$V_{Cu} = \frac{m}{8_{Cu}}$, $V_{Zn} = \frac{12-m}{8_{Zn}}$

$V = V_{Cu} + V_{Zn}$

$= \frac{m}{8_{Cu}} + \frac{12-m}{8_{Zn}}$

$= \frac{m}{8} + \frac{12-m}{7}$

$U = V\sigma g$

$V = \frac{U}{\sigma g} = \frac{m}{8} + \frac{12-m}{7}$ $\qquad \frac{(12-11)g}{\sigma g} = \frac{1}{\sigma} = 1$

total vol. ; σ of water

so, $1 = \frac{m}{8} + \frac{12-m}{7}$

or, $56 = 7m + 96 - 8m$

or, $m = 96 - 56$

$\therefore \boxed{m = 40gm}$

Concept

A certain block weighs 15N in air. It weighs 12N in water. When immersed in another liquid, it weighs 13N. Calculate the relative density of liquid.

→ $R.D = \frac{\rho_b}{\rho_w}$

for body

$R.D = \frac{\text{wt. of body}}{\text{wt. of equal vol. of water}}$

$= \frac{W_{body}}{\text{Upthrust}} = \frac{W_A}{W_A - W_w}$

$= \frac{15}{15-12} = 5.$

Relative density of liquid = ?

For liq,

$R.D = \frac{\text{Upthrust in liq.}}{\text{Upthrust in water}}$

$= \frac{W_a - W_{liq}}{W_a - W_{water}}$

$= \frac{15-13}{15-12} = \frac{2}{3}$

Floatation

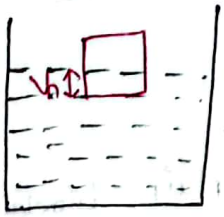
① $\rho_{body} < \sigma$ → Float partially

② $\rho_{body} = \sigma$ → Float (fully submerged)



③ $\rho_{body} > \sigma$ → sink

* Fraction of volume outside the liquid for floating body



$W = U$

$V \cdot \rho g = V_{in} \sigma g$

$$V_{in} = \left(\frac{\rho}{\sigma}\right) V$$

$V_{outside} = V - \left(\frac{\rho}{\sigma}\right) V$

$$V_{outside} = \left(1 - \frac{\rho}{\sigma}\right) V$$

Fraction

$$F_{out} = \frac{V_{outside}}{V} = \left(1 - \frac{\rho}{\sigma}\right)$$

Q. For a floating body, what is the fraction of outside if the density of sea water is 1.03gm/cc.
(density of ice = 0.9gm/cc)

$$F_{out} = \frac{V_{out}}{V} = \left(1 - \frac{\rho}{\sigma}\right) = \left(1 - \frac{0.9}{1.03}\right)$$

=

Comparison of densities of Floating body

For Floating

$$W = U$$

$$V\rho = V_{in}\sigma$$

$$\boxed{\rho = \frac{V_{in}}{V}\sigma}$$

If different bodies are floating in same liquid :-

$$\cancel{\frac{\rho_1}{\rho_2} = \left(\frac{f_{in}}{f_{out}}\right)_2^1} \qquad \boxed{\frac{\rho_1}{\rho_2} = \frac{(f_{in})_1}{(f_{in})_2}} \qquad \rho \propto f_{in}.$$

→ If $\rho_1 > \rho_2$, $(f_{in})_1 > (f_{in})_2$.

comparison of densities of liquids

If same body is made to float in different liquid,

Then

$$W = U$$

$$V\rho = (V_{in})_1\sigma_1 = (V_{in})_2\sigma_2$$



$$\boxed{\frac{\sigma_1}{\sigma_2} = \frac{(V_{in})_2}{(V_{in})_1}}$$

[Density धेरै हुने liquid मा body धेरै inside जान्छ।]

→ A piece of ice is floating in water. What will happen the level of water when all ice melts?

→ What will happen when the beaker is filled with any liquid (except water)?

(i) Denser than water (ii) Lighter than water

Concept

$M \rightarrow$ mass of ice floating in liquid of density ρ_l.

* wt. of ice = upthrust

$$Mg = V_{displ.} \rho_l g$$

$V_{displ.}$ → vol. of displaced liquid.

- Volume of displaced liquid

$$V_D = \frac{M}{\rho_l}$$



- When ice melts, (M same)

$\rho_w \rightarrow$ density of water

Volume of water formed, $V_F = \frac{M}{\rho_w}$

- If the liquid is water, $(\rho_l = \rho_w)$

$$V_D = \frac{M}{\rho_w}$$

$\therefore \boxed{V_D = V_F} \rightarrow$ level remains same

- For any liquid →

(i) $\rho_l > \rho_w \rightarrow$ denser than water

$$V_D = \frac{M}{\rho_l} \qquad V_F = \frac{M}{\rho_w}$$

$$\frac{M}{\rho_l} > \frac{M}{\rho_w}$$ हुन भएन

$$\frac{M}{\rho_l} < \frac{M}{\rho_w} \Rightarrow V_D < V_F$$

↓

level rises.

(ii) $\rho_l < \rho_w$

$$\frac{M}{\rho_l} > \frac{M}{\rho_w}$$

$\therefore V_D > V_F$ → level decreases / falls.

conclusion

① Ice melt
- If liquid is water → level same ($V_D = V_F$)
- If liquid is heavier than water → level rises ($V_D < V_F$)
- If liquid is lighter than water → level falls ($V_D > V_F$)

concept

A beaker exactly full of water has an ice piece floating on it. As the ice cube melts, what happens to the water level.

(i) If cube contains air bubble

(ii) If cube contains
- pb (lead or stone)
- cork

→

M → Mass of ice floating

$M = V_D \sigma_w$

$$V_D = \frac{M}{\sigma_w}$$

If cube contains air which is weightless

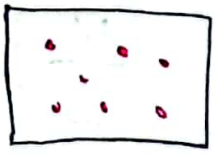
M gram of ice melts (M same)

. Volume of water formed by melting ice

$$V_F = \frac{M}{\sigma_w}$$

$\therefore V_D = V_F$ → level same.

If ~~cube~~ M mass of ice contains embadded piece of mass m



~~Mg = V~~

For Floating

$$(M+m)g = V_D \sigma_w g$$

$$V_D = \frac{M}{\sigma_w} + \frac{m}{\sigma_w}$$

when ice melts

water formed will be of mass 'M'.

$$(V_F)_1 = \frac{M}{\sigma_w}$$

M ice melts ⇒ get volume of water.

(i) If the ice cube contains (lead piece)

$\rho_{Pb} > \rho_w$ (lead sink)

↓

it displaces water equal to its own volume (lead volume)

$$V_2 = \frac{m}{\rho_{Pb}}$$

after melting ice cube

Volume added $V = (V_F)_1 + V_2$

$$V = \frac{M}{\sigma_w} + \frac{m}{\rho_{Pb}}$$

$\because \rho_{Pb} > \rho_w$

$\therefore V_D > V$ → level falls.

$\rho_{cork} < \rho_w$ → cork will float

↳ cork माथि आउछ

when ice melt

water displaced by cork,

$$mg = V_2' \rho_w g$$

$$V_2' = \frac{m}{\rho_w}$$

$$V = V_1 + V_2$$

$$V = \frac{M}{\rho_w} + \frac{m}{\rho_w}$$

$\therefore$ $V_D = V$ → level same

Conclusion

Ice Floating in water →

when melts → level same.

If ice cube contains cork (lighter than water)

↓

when melts → level same

If ice cube contains lead → (पानीमा डुब्ने)

↓

when melts → level falls.

Abishek singh

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